RF/Photonics Lab
Michael Benker
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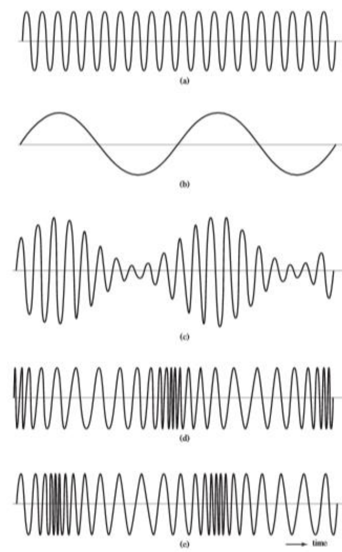
Angle Modulation

In comparison to Amplitude Modulation, which varies the magnitude of the sinusoidal carrier wave, Angle Modulation varies the phase of the carrier wave. The two most common forms of angle modulation are phase modulation (PM) and frequency modulation (FM). Phase modulation varies the instantaneous angle linearly with the message signal, while frequency modulation varies the instantaneous frequency with the message signal.

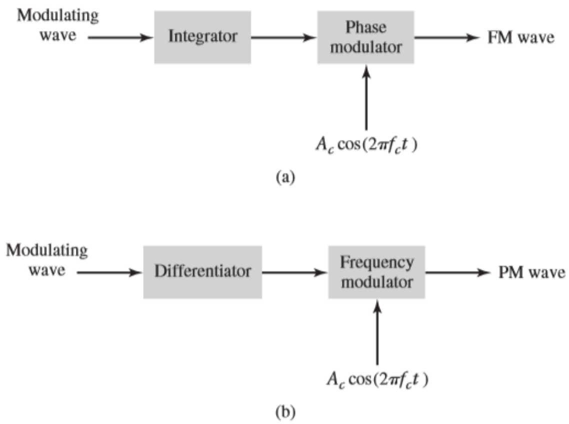


The signals on the right are understood (from top to bottom) as the carrier frequency, the modulating wave and the result signal of amplitude modulation, phase modulated and frequency modulation. Due to phase modulated and frequency modulated waves having constant amplitude A_C, noise is expected to be lower, although the transmission bandwidth is increased. Rates of distortion are reduced with a reduced possibility of a polarity shift. The average power for angle modulated wave is $P_{ave}=(1/2)*(A_C)^2$.

The table below summarizes the relationship between phase-modulated and frequency-modulated waves. An FM wave can be seen as a PM wave with a substitution of the integral of the message signal for the message signal. Further, an FM wave can be represented as having gone through an *integrator* while a PM wave is represented as having gone through a *differentiator*.

TABLE 4.1 ***Summary of Basic Definitions in Angle Modulation***

	Phase modulation	*Frequency modulation*	*Comments*
Instantaneous phase $\theta_i(t)$	$2\pi f_c t + k_p m(t)$	$2\pi f_c t + 2\pi k_f \int_0^t m(\tau)\,d\tau$	A_c: carrier amplitude f_c: carrier frequency $m(t)$: message signal k_p: phase-sensitivity factor k_f: frequency-sensitivity factor
Instantaneous frequency $f_i(t)$	$f_c + \frac{k_p}{2\pi}\frac{d}{dt}m(t)$	$f_c + k_f m(t)$	
Modulated wave $s(t)$	$A_c \cos[2\pi f_c t + k_p m(t)]$	$A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(\tau)\,d\tau\right]$	

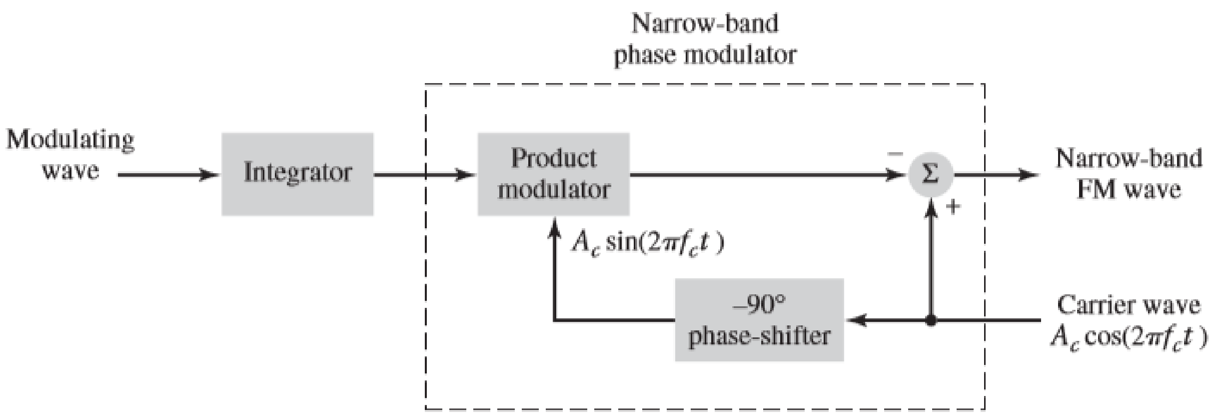


The benefits of conserving bandwidth lead to the development of the **narrow-band frequency modulation** scheme. To achieve this, several parameters are defined. The frequency deviation, or the maximum departure of the instantaneous frequency from the carrier frequency is defined as $\Delta f = k_f A_m$, where k_f (as mentioned in Table 4.1) is the frequency sensitivity factor. The modulation index, β is the ratio of the frequency deviation to the modulation frequency: $\beta = \Delta f / f_m$. The angle of the FM wave and the FM wave itself are described as:

$$\theta_i(t) = 2\pi f_c t + \beta \sin(2\pi f_m t)$$

$$s(t) = A_c \cos[2\pi f_c t + \beta \sin(2\pi f_m t)]$$

The following block diagram depicts a method for generating a narrow-band FM wave:



Carson's rule defines an approximate relation for the transmission bandwidth of an FM wave generated by a single-tone modulating wave. From the following expression (Carson's rule), it is understood that large values of the modulation index β the bandwidth is slightly greater than the twice the frequency deviation Δf and for small values of the modulation index, the spectrum is limited to the carrier frequency and a pair of side-frequencies at $f_c \pm f_m$, in which case the bandwidth approached $2*f_m$.

$$B_T \approx 2\Delta f + 2f_m = 2\Delta f\left(1 + \frac{1}{\beta}\right)$$